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# Error Gradients in Neural Networks

## Problem 1

Consider the neural network illustrated in Fig. 1, with inputs $x_1 = 3.1$ and $x_2 = 2.7$. The output neuron has a binary sigmoidal function. The weights are $w_0 = 0.6$, $w_1 = -0.5$, and $w_2 = 0.5$. The desired output of the network is $d = 0.9$, and the actual output is denoted by $s$.

(a) Calculate the error $e = d - s$ and the half square error $\mathcal{E} = \frac{1}{2} e^2$ in the network response.

(b) Calculate the gradients of $\mathcal{E}$, defined in part (a), with respect to the weights $w_0$, $w_1$, and $w_2$, respectively, at the weight values given.

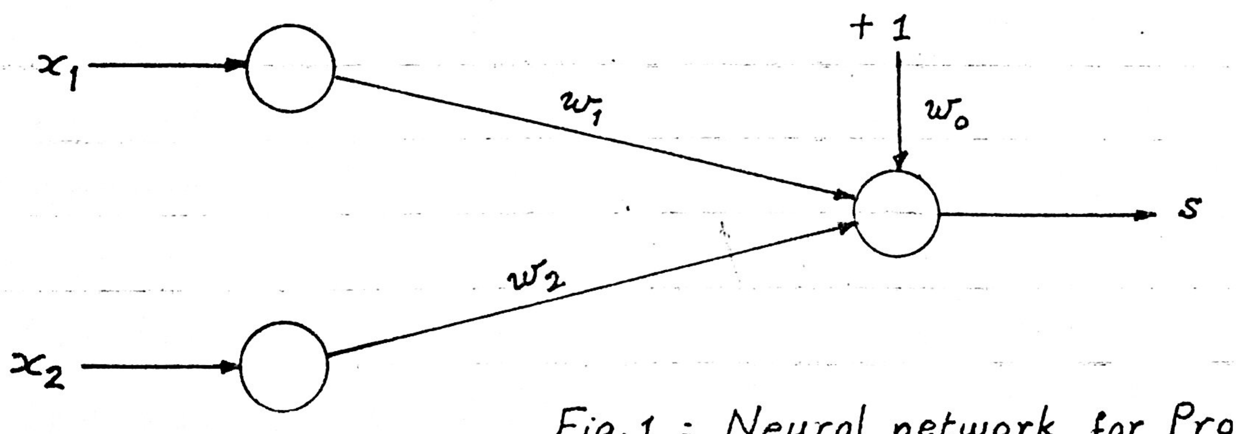


Fig. 1 : Neural network for Prob. 1

## Problem 2

Repeat Prob. 1 when the output neuron has a

bipolar sigmoidal function.

## Problem 3

Refer to the neural network of Fig. 1. The gradients of $\mathcal{E}$, defined in Prob. 1, with respect to weights $w_0$, $w_1$, and $w_2$, respectively, are found to be

$$\frac{\partial \mathcal{E}}{\partial w_0} = -0.13, \quad \frac{\partial \mathcal{E}}{\partial w_1} = -0.52, \quad \frac{\partial \mathcal{E}}{\partial w_2} = -0.65$$

at particular weight values. Determine the inputs $x_1$ and $x_2$.

## Problem 4

In Prob. 3, let $w_1 = 0.5$ and $w_2 = -0.5$. Calculate the value of $w_0$ if $e$, defined in Prob. 1, is found to be $-0.52$. The output neuron has a binary sigmoidal function.

## Problem 5

Repeat Prob. 4 if $e$ is found to be $0.65$.

## Problem 6

Can you solve Prob. 4 with $e = 0.5$? Justify.

## Problem 7

Consider the neural network illustrated in Fig. 2,

with inputs $x_1 = -2.6$ and $x_2 = -1.9$. The two output neurons have binary sigmoidal functions. The weights are :

$$w_{11} = 1.1 \;, \quad w_{12} = -1.2 \;, \quad w_{21} = -0.8 \;,$$
$$w_{22} = 0.7 \;, \quad w_{01} = 0.35 \;, \quad w_{02} = -0.46$$

The desired outputs of the network are $d_1 = 0.51$ and $d_2 = 1.21$, and the corresponding actual outputs are denoted by $s_1$ and $s_2$, respectively.

(a) Calculate the errors $e_1 = d_1 - s_1$ and $e_2 = d_2 - s_2$, and the half summed square error $\mathcal{E} = \frac{1}{2}(e_1^2 + e_2^2)$.

(b) Calculate the gradients of $\mathcal{E}$, defined in part (a), with respect to the weights of the network, respectively, at the weight values given.

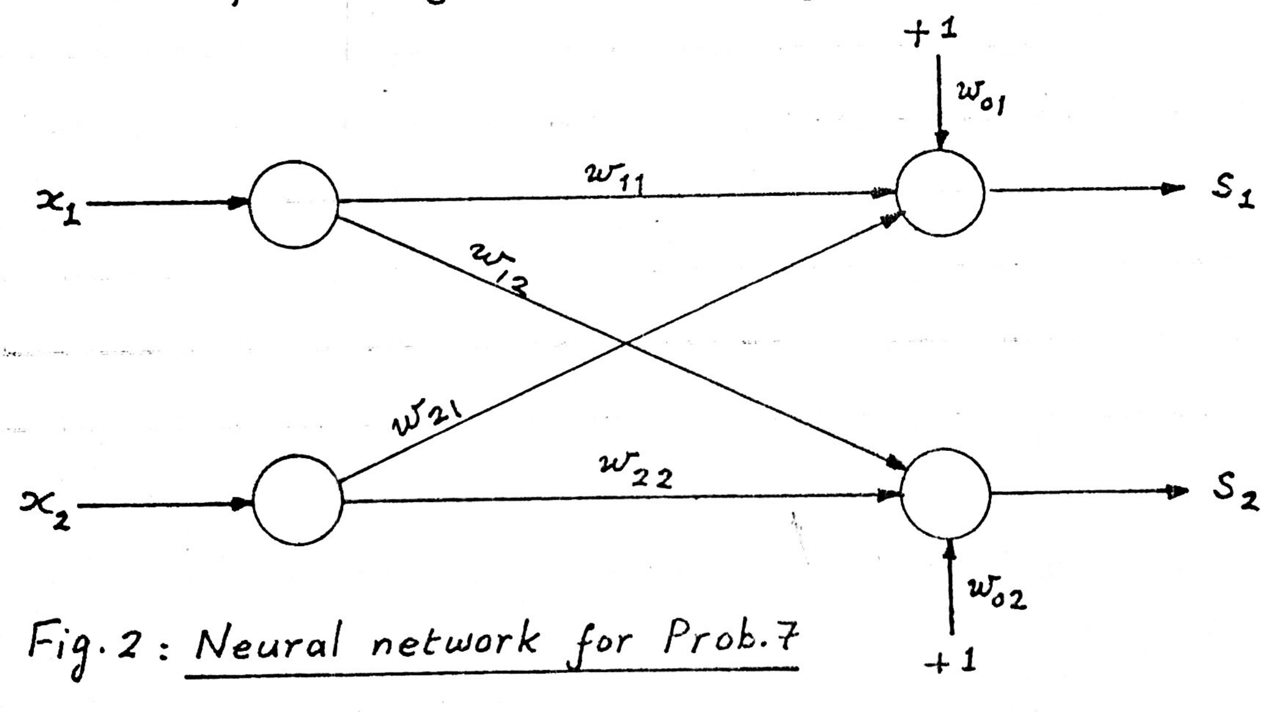


Fig. 2 : Neural network for Prob. 7

## Problem 8

Refer to the neural network of Fig. 2. At output neuron 1, the error $e_1 = d_1 - s_1$ is 1.22 when the

desired output $d_1$ is 1.41. At output neuron 2, the error $e_2 = d_2 - s_2$ is $-0.81$ when the desired output $d_2$ is $-1.23$. The two output neurons have bipolar sigmoidal functions. The weights are:

$w_{11} = -0.92$, $w_{12} = 0.87$, $w_{21} = 0.76$,

$w_{22} = -0.65$, $w_{01} = -0.53$, $w_{02} = 0.53$

(a) Determine the inputs $x_1$ and $x_2$.

(b) Calculate the gradients of the half summed square error $\mathcal{E} = \frac{1}{2}(e_1^2 + e_2^2)$ with respect to the weights of the network, respectively, at the weight values given.

## Problem 9

Refer to the neural network of Fig.2. The output neurons have bipolar sigmoidal functions. The errors in the response signals $s_1$ and $s_2$ are $-0.8$ and $-0.9$, respectively, for particular values of network inputs $x_1$ and $x_2$. The weights are:

$w_{11} = -1.21$, $w_{12} = 1.44$, $w_{21} = 0.88$,

$w_{22} = -0.75$, $w_{01} = -0.4$, $w_{02} = -0.4$

The gradients of the half summed square error $\mathcal{E}$ with respect to $w_{01}$ and $w_{02}$ are found to be $\frac{\partial \mathcal{E}}{\partial w_{01}} = 0.3$ and $\frac{\partial \mathcal{E}}{\partial w_{02}} = 0.288$. Calculate the gradients of $\mathcal{E}$ with respect to the remaining weights, respectively. What are the values of inputs $x_1$ and $x_2$?

## Problem 10

Consider the neural network illustrated in Fig. 3. The two output neurons have binary sigmoidal functions. The response signals are $s_1 = 0.6$ and $s_2 = 0.7$, with errors $e_1 = 0.35$ and $e_2 = -0.35$, respectively. The three inputs are constrained with a condition $x_1 = -x_2 = 2x_3$, and the two bias weights are constrained with a condition $w_{01} + w_{02} = 1$. The weights of the connections between input and output neurons are:

$w_{11} = -0.5$, $w_{12} = -0.5$, $w_{21} = 0.4$,

$w_{22} = 0.3$, $w_{31} = 0.2$, $w_{32} = -0.6$

(a) Determine the inputs $x_1$, $x_2$, and $x_3$, and the bias weights $w_{01}$ and $w_{02}$.

(b) Calculate the gradients of the half summed square error with respect to all weights of the network at the weight values specified.

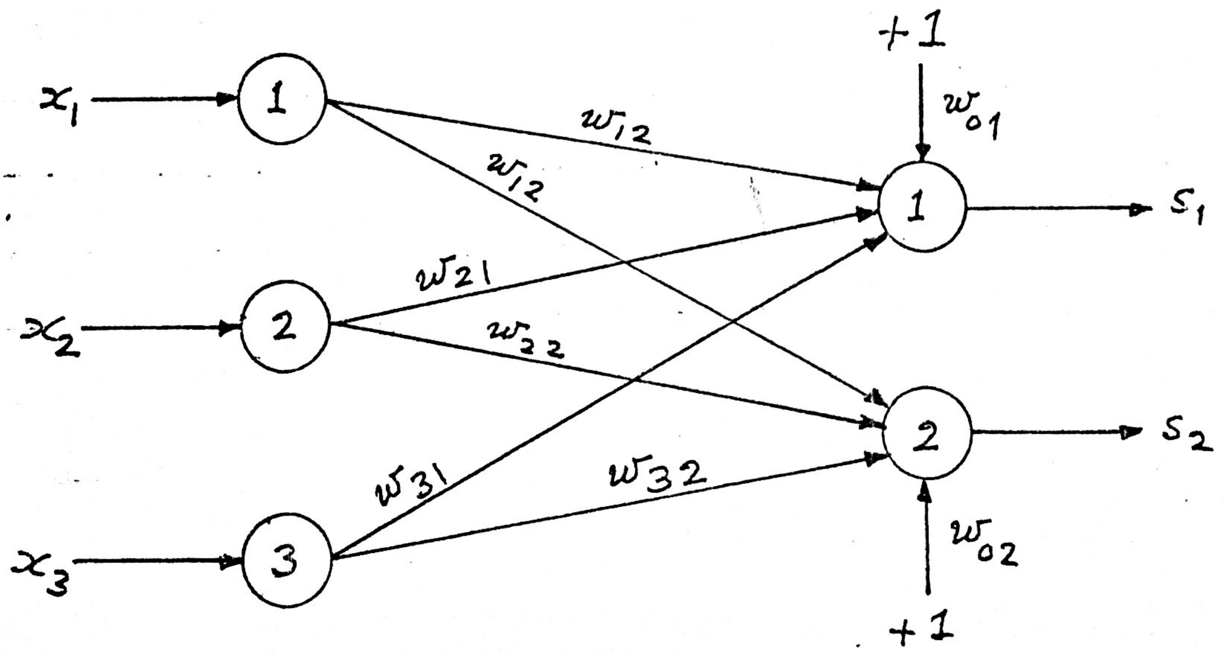


Fig. 3: Neural network for Prob. 10

## Problem 11

Consider the neural network illustrated in Fig. 4. The neurons of both the hidden and output layers have binary sigmoidal functions. The inputs are $x_1 = 2.2$ and $x_2 = -1.8$. The hidden-to-output weights are:

$$w_{11} = 0.65 \quad , \quad w_{21} = -0.55 \quad , \quad w_{01} = 0.4$$

and the input-to-hidden weights are:

$$\overline{w}_{11} = 0.7 \quad , \quad \overline{w}_{12} = 0.7 \quad , \quad \overline{w}_{21} = 0.6 \, ,$$
$$\overline{w}_{22} = 0.6 \quad , \quad \overline{w}_{01} = 0.5 \quad , \quad \overline{w}_{02} = -0.5$$

Calculate the gradients of the half square error with respect to all weights of the network, respectively, at the weight values given. Assume the desired response of the network to be of value 0.9.

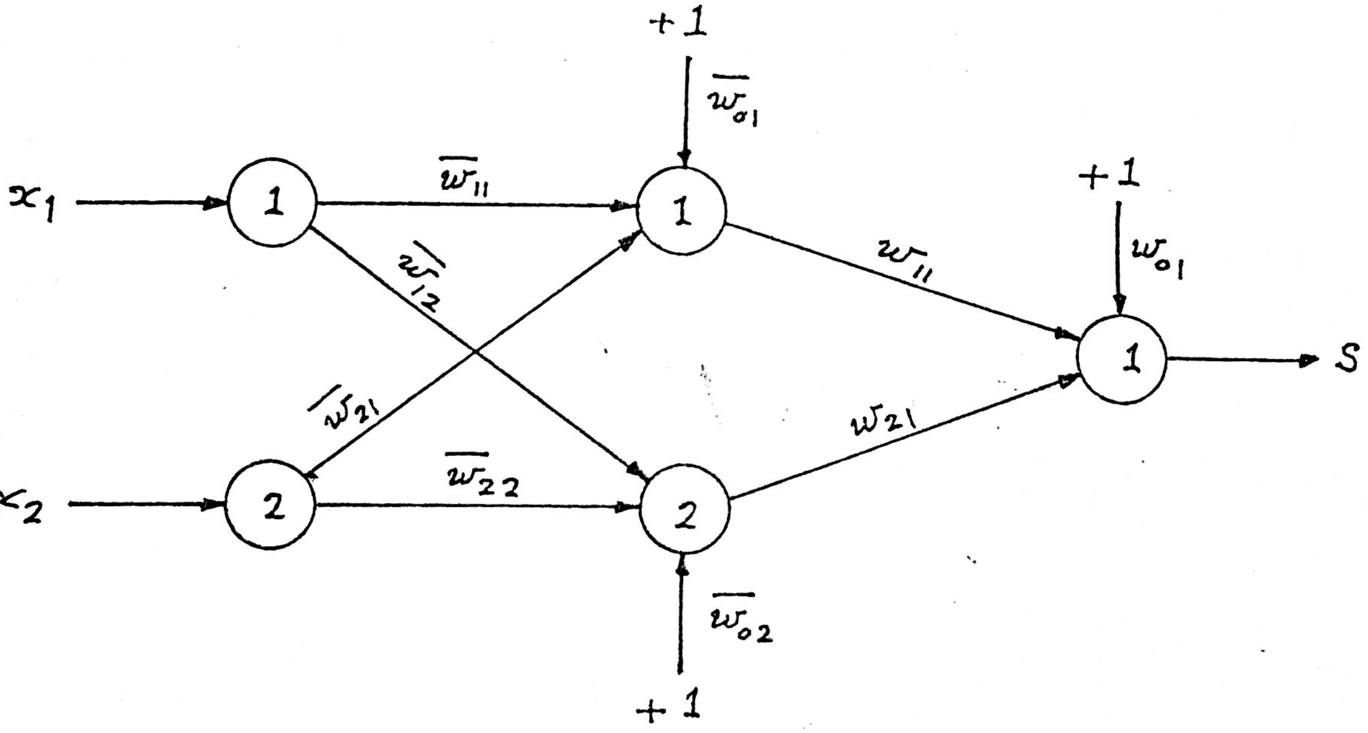


Fig. 4 : Neural network for Prob. 11

## Problem 12

Consider the neural network illustrated in Fig. 5. The neurons of both the hidden and output layers have binary sigmoidal functions. The inputs are $x_1 = -3.2$ and $x_2 = 2.8$. The actual response signals are denoted by $s_1$ and $s_2$, and the corresponding desired signals are 0.18 and 0.11, respectively. The hidden-to-output weights are:

$$w_{11} = 0.7, \quad w_{12} = 0.7, \quad w_{21} = 0.6, \quad w_{22} = 0.6,$$
$$w_{01} = 0.5, \quad w_{02} = 0.4$$

and the input-to-hidden weights are:

$$\overline{w}_{11} = 0.85, \quad \overline{w}_{12} = -0.9, \quad \overline{w}_{21} = 0.77, \quad \overline{w}_{22} = -0.9,$$
$$\overline{w}_{01} = -0.6, \quad \overline{w}_{02} = 0.62$$

Calculate the gradients of the half summed square error with respect to all weights, respectively, at the weight values given.

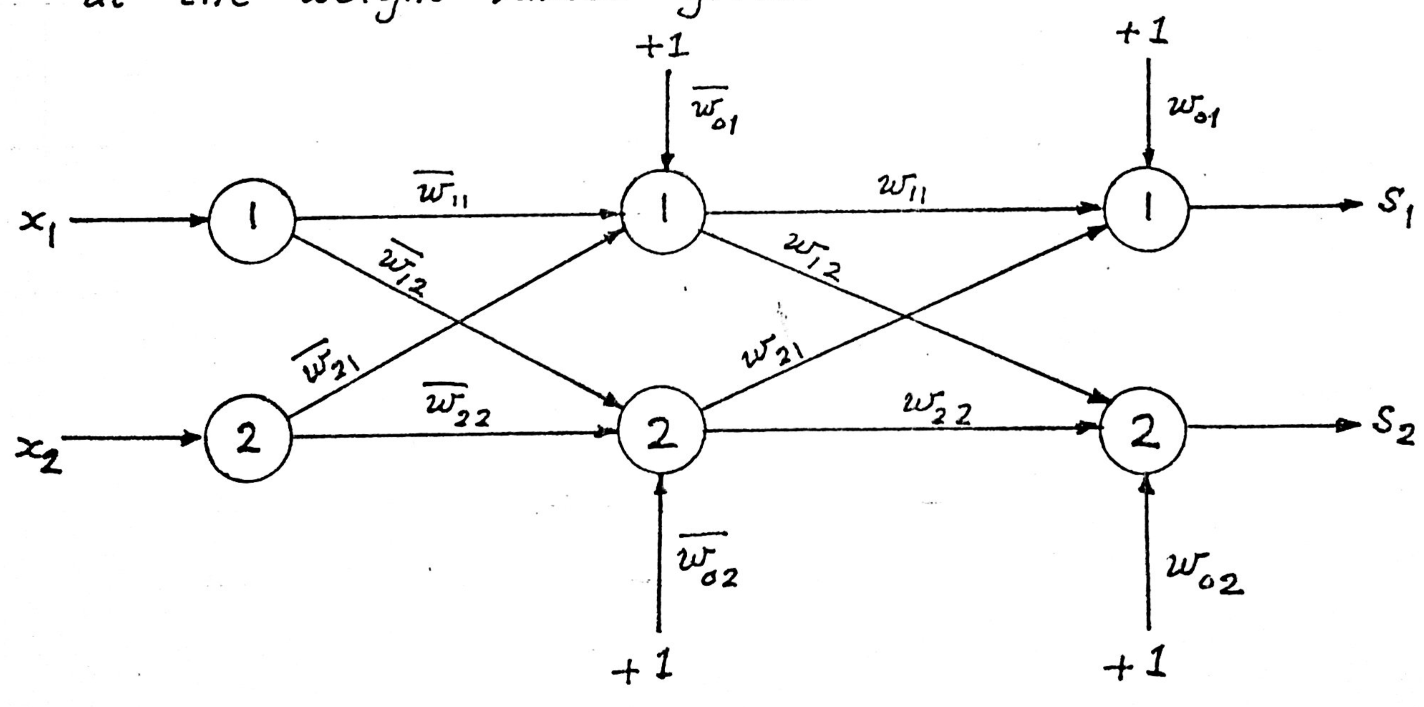


Fig. 5 : Neural network for Prob. 12